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REVIEWS

Propagation of a Curved Shock and Nonlinear Ray Theory. By P. PRASAD. Longman, 1993. 134 pp. £22. ISBN 0582 07253 0.

This volume summarizes recent developments in the inviscid theory of shock waves with main emphasis on work performed at the Indian Institute of Science, Bangalore. Much of the material is devoted to the outline of a new theory of shock dynamics (NTSD). The key element of this theory is the observation that the motion of a shock surface in space can be described by means of an equation which has the same general form as the equation which governs the motion of characteristic surfaces. As a result, it is possible to define shock rays and to derive compatibility conditions on the shock surface. The study of the latter ones, however, reveals an essential difference between wave and shock fronts. While a wave front is self-propagating in the short wave limit, e.g. it can be calculated if the distribution of the field variables on the surfaces is known at some time, this is not true for a shock front. Of course, this reflects the fact that the motion of a shock front is not fully determined by the initial data on the front but depends on the disturbances merging from upstream and downstream as well. In mathematical terms this means that an infinite rather than a finite set of compatibility conditions is needed to calculate successive positions of a shock surface. Truncation of the infinite set at the nth stage then leads to a problem of solving only a finite number of equations, which the author calls a NTSD.

Following a brief introduction to the theory of shock waves which addresses the roles of conservation laws, jump conditions and entropy conditions the NTSD is developed first for the most simple case of a single conservation law in one space dimension. Exact solutions of the Riemann equation are compared with numerical solutions based on the infinite set of compatibility conditions truncated at various stages ranging from $n = 2$ up to $n = 25$. The comparison indicates that reasonable accuracy can be achieved by integrating only four or five equations. Moreover, unlike other finite difference methods the shock remains a sharp discontinuity and is not spread over a few grid points.

As shown by the author the NTSD can be readily extended to one-dimensional problems involving a set of conservation laws. As a specific example the gasdynamic piston problem is investigated. Comparison of the results with numerical solutions obtained by means of McCormack's scheme again indicates that accurate predictions can be made for moderately large time using a relatively small system of compatibility conditions only if the piston path is sufficiently smooth and does not generate multiple shocks or shock-fan combinations.

The final chapters of the book are devoted to the generalization of the NTSD to three-dimensional problems where a new non-trivial element, e.g. the rotation of wave or shock fronts along rays, enters the mathematical description. Following general considerations for arbitrary systems of hyperbolic conservation laws gasdynamic flows are treated as an example. For small-amplitude waves a set of transport equations is formulated which differs from the results of classical nonlinear geometric acoustics insofar as the rotation of wave fronts along rays is taken into account in the first-order approximation. On the basis of analytical considerations and preliminary numerical results it is argued that this set of equations also remains valid in situations where classical geometric acoustics breaks down, as for example in a caustic region. Further efforts appear to be necessary, however, to clarify this important point.

In conclusion, this volume gives a full picture of a new approach to shock dynamics which has been pursued intensively by the author and his colleagues in the past decade. Unfortunately, references to other work are sometimes less satisfactory. Specifically, the discussion of Whitham's theory of shock dynamics would certainly have benefited by mentioning the warning included in the original paper that the 'approximate method is most suitable for strong shocks with Mach number greater than about 2, and must be used with care for weaker ones'. Also, it would have been helpful to indicate that the importance of wave-front turning effects in general and in particular in the vicinity of singular rays has been recognized by various authors in the past. Indeed, a critical comparison of the NTSD with the asymptotic theory of singular rays leading to the Zabolotskaya Khokhlov equation would undoubtedly have increased the impact of the book, as would a few comments on the limitations of inviscid theory and the effects of finite shock thickness.

The book should be of interest to readers working in nonlinear wave theory and in particular in the area of nonlinear acoustics.

A. KLUWICK

Vortex Structures in a Stratified Fluid. By S. I. VOROPAYEV and Y. D. AFANASYEV. Chapman & Hall, 1994. 230 pp. £34.50.

The subtitle of this book, 'Order from chaos', immediately indicates the authors' underlying motivation for studying the phenomena presented. Their introductory section gives many geophysical examples of meso-scale and synoptic coherent structures which are a feature of the ocean surface and an essential component of all two-dimensional turbulent flows. The scale of these vortex structures lies between that of the ocean-wide circulation patterns and the small-scale turbulent motions. The authors argue that because there is a spectral energy gap near this scale, it is fruitful to treat the vortices as entities, having several elementary forms, which are acted on by an effective viscosity due to the small-scale turbulence. This turbulent viscosity is large, so the effective Reynolds number of these structures is only moderate, and it is reasonable to model them in the laboratory.

The presentation in this volume is unusual in its combination of simple laboratory experiments and a careful exposition of the related theory. Its level is somewhere between that of a graduate student textbook and a research monograph. The latter description is suggested by the choice of topics, which are closely related to the authors' current research papers, some of which have only recently been published. The first third or more of the book (the first two chapters, and part of the third) presents the background needed to make the exposition nearly self-contained, without the need to refer to other textbooks.

Chapter 1 sets out the experimental techniques (which are simple enough to be learnt and used by interested readers), and gives examples of the impulsively generated and continuous flows to be considered later. These are illustrated by photographs of high quality, which are used prolifically and incorporated into the text throughout the book. Two techniques are used to produce two-dimensional viscous flows. In the first, the flows are induced in a thin layer of fluid that is floated on a deeper, denser layer which is less viscous, and may be miscible or immiscible with the upper fluid. The second technique is to produce a linear salinity gradient and inject dyed salt solution at its own density level. A graphic example is given of a horizontal turbulent jet injected impulsively, rapidly forming a coherent flat vortex dipole in which all the small-scale motions have died away. In the theoretical chapter 2, the authors derive the governing equations using vector and tensor notation, first in three dimensions and then specializing to planar flows. They introduce the concepts of point forces and force dipoles, and develop multipole expansions, valid at a large distance from the source, for the stream function in two dimensions. They also discuss self-similarity at some length, a concept which is used to describe many of the flows of interest. Finally, they treat density-stratified flows, and give a clear physical and mathematical analysis of the form of the horizontal and vertical velocity profiles in a submerged horizontal jet, and its approach to a quasi-two-dimensional state. It is shown that the Richardson number for these unsteady flows rapidly becomes large and the motion is horizontal. Thus two-dimensional models can be used to describe the resulting flows analytically, and some new solutions are presented later.

The theory of two-dimensional motions induced in a homogeneous viscous fluid by localized distributions of vorticity is presented systematically in chapter 3, and here there are fewer experimental comparisons. Standard solutions are presented for the streamline patterns of a point-vortex couple and a distributed vortex dipole, with a discussion of the impulse vector, and the relation between this and flow momentum. The solution for the diffusion of a line vortex is extended to include combined sources of vorticity and mass. Stokes dipoles produced by point forces or momentum sources (i.e. impulsive or continuous jets), are treated in the linear approximation, and the solutions are shown to be in reasonable agreement with observation. Two dipoles of opposite sign, produced by jets of either of the above kinds placed close together and directed away from one another, or by the force dipole set up by a small oscillating cylinder, create vortex quadrupoles. The nonlinear extensions to the quadrupole solutions are also considered at some length, and it is shown that, to second order in the Reynolds number, the flows at large times are steady and consist of narrow outflowing jets in the direction of the oscillation, with broader inflows between. These results also agree with the experimental flows described later.

The last two chapters present the synthesis the authors have been leading up to: the experimental observations of vortex dipole interactions and their interpretation in terms of the forces acting on the fluid. The 'elementary particle' of two-dimensional chaotic flows is the vortex dipole, and combinations of these, colliding and interacting in various geometries, can explain a whole range of complex phenomena. Symmetric head-on collisions of vortex dipoles produce a vortex quadrupole, developing as if from a point source. Collision of impulsive jets with different momenta gives unsymmetric vortices with residual rotation, and a range of behaviours which depends sensitively on the ratio of the initial momenta. The merging of two dipoles, one behind the other, and also those moving on parallel or oblique paths, are calculated in turn by superimposing the appropriate linear solutions, and the results are shown to compare well with the corresponding laboratory observations. Non-axial interactions, with the formation of an unsteady rotating quadrupole, and the interaction with solid walls and small bodies are also treated, with clear physical descriptions in terms of the primary processes and their interactions.

The last chapter outlines a more empirical approach to the development of vortex structures, using integral relations and the parameterization of the external fluxes of mass and other conserved quantities into the moving volume. Similarity solutions are obtained, based on mechanistic arguments and entrainment hypotheses similar to those which are familiar in the context of buoyant convection, for both a horizontal 'starting jet' in a linearly stratified viscous fluid and an impulsively generated vortex dipole. For the latter flow, calculations are also presented of the trajectories of tracer particles relative to the expanding boundary of the vortex, following previous kinematic models for inviscid and viscous buoyant 'thermals'.

The final phenomenon treated in some detail is the developing vortex quadrupole formed by oscillating a thin rod in a viscous fluid. The limiting behaviours for small and moderate Reynolds number are treated separately and compared with recent experiments by the authors and their colleagues; there is a lot of empiricism here in making the comparison. The results for a single quadrupole are extended to a linear array of quadrupoles, located part way across an experimental tank far from the walls or right across the tank. This last geometry is of course relevant to understanding of turbulence generated by an oscillating grid of thin parallel rods, when the individual dipoles grow and interact as they propagate away from the grid. Dimensional arguments, plus some *ad hoc* assumptions suggested by comparison with measurements of the width of the expanding turbulent region as a function of time, suggest that the intensity of forcing is the main governing parameter of this flow, and that the spacing of the rods plays only a minor role. The arguments are extended to describe the three-dimensional turbulence produced by a grid of crossed wires or bars, and compared with Long's formulation and interpretation of experiments in terms of the 'action' K. By definition K is proportional to Q/ν, where Q is the strength of the quadrupole and ν the kinematic viscosity. Note that the authors' theory relates to the case of thin bars or wires, at which the streaming flow produced depends explicitly on viscosity, and the velocity v is predicted to be proportional to $f^{3/2}$, where f is the frequency of oscillation. For the configuration which is commonly used to produce the turbulence in experiments on mixing across density interfaces, the result is quite different. With larger grid bars of square cross-section and large amplitudes of oscillation, v is proportional to f, suggesting that the velocity of the separating flow is directly proportional to the maximum grid velocity, with no direct dependence on ν.

Though the scope of this book is determined closely by the authors' recent research interests, so that it might therefore be regarded as rather narrow, I found it a very useful exposition of ideas which I had known about in otherwise unrelated contexts. The relationship they have brought out between interacting eddies in two-dimensional turbulence and flows generated by oscillated bodies is straightforward but not obvious, and it does help to have them treated in such a coherent and logical manner. The standard of the writing and production is good, and I noted only one substantial misprint – the omission of equations (25) and (26) on page 115 and the repetition instead of (27) and (28). The neat experimental methods developed to illustrate the flows and make graphic comparisons with theory also make this a visually appealing book which I can recommend to anyone interested in obtaining a clear physical understanding of vortex dynamics.

J. S. TURNER